Schrödinger equation - by [ˈɛʁ̯viːn ˈʃʁøːdɪŋɐ] (en: /ˈʃroʊ-dɪŋər/)

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full equation: $i\hbar\frac{\partial}{\partial t}\Psi(r,t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(r,t) + V(r,t)\Psi(r,t)$

Time-independant equation for one dimension: $E\mathit{\Psi}(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial^2 x}\mathit{\Psi}(x) + E_{pot}\mathit{\Psi}(x)$

Some useful equations: $E = E_{kin} + E_{pot}$ $E_{kin} = \frac{1}{2}mv^2$ $k = \frac{2\pi}{\lambda} = \frac{p}{\hbar}$ $p = mv$ $e^{ix} = \cos(x) + i\sin(x)$

Remember, energy times our wave is kinetic energy plus potential energy, we can do this because quantum mechanics is fun and quirky and not confusing at all: $E\mathit{\Psi} = E_{kin}\mathit{\Psi} + E_{pot}\mathit{\Psi}$

Another way to show wave function: $\mathit{\Psi}(x,t) = A\cos(kx - \omega t)$

We add imaginary component to make math easier: $\mathit{\Psi}(x,t) = A(\cos(kx - \omega t) + i\sin(kx - \omega t))$

Euler formula comes to the rescue: $\mathit{\Psi}(x,t) = Ae^{i(kx-\omega t)}$

Wave equation can describe our wave: $\frac{\partial^2}{\partial^2 x}\mathit{\Psi} = \frac{1}{c^2}\frac{\partial^2}{\partial^2 t}\mathit{\Psi}$

So lets find the second derivative of our wave: $\frac{\partial^2}{\partial^2 x}\mathit{\Psi} = \frac{\partial^2}{\partial^2 x}Ae^{i(kx-\omega t)}$

The result: $\frac{\partial^2}{\partial^2 x}\mathit{\Psi} = -k^2\mathit{\Psi}$

Lets simplify k^2: $\frac{\partial^2}{\partial^2 x}\mathit{\Psi} = -\frac{p^2}{\hbar^2}\mathit{\Psi}$

Remember E_kin = 1/2*m*v^2? We can rearrange as this because velocity is momentum over mass: $E_{kin} = \frac{1}{2}\frac{mp^2}{m^2} = \frac{p^2}{2m}$

Because of this: $p^2 = 2mE_{kin}$

Lets plug that in: $\frac{\partial^2}{\partial^2 x}\mathit{\Psi} = -\frac{2m}{\hbar^2}E_{kin}\mathit{\Psi}$

Lets rearrange: $\frac{\partial^2}{\partial^2 x}\mathit{\Psi} \cdot -\frac{\hbar^2}{2m} = E_{kin}\mathit{\Psi}$

Lets plug that in: $E\mathit{\Psi} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial^2 x}\mathit{\Psi} + E_{pot}\mathit{\Psi}$