Lesson 3 – Hyperoperators / Knuth's up-arrow notation

This presentation should help you understand:
2.2.

Presentation made by Mr. A, for a googology canvas course.

What is after exponentiation?

- Consider that addition is repeated succession (counting), multiplication is repeated addition, and exponentiation is repeated multiplication. What would be the next logical step?
- Tetration! Which is repeated exponentiation.

$$n \uparrow\uparrow x = \overbrace{n^{n^{\cdot^{\cdot^{\cdot^{n}}}}}}^{x}$$

- With tetration we can define numbers that are much larger; for instance, $10 \uparrow\uparrow 10$ is already significantly larger than the Poincare recurrence time.

What's next?

- Well next is pentation:

$$n \uparrow\uparrow\uparrow x = \overbrace{n \uparrow\uparrow n \uparrow\uparrow n \ldots \uparrow\uparrow n \uparrow\uparrow n}^{x}$$

- After that its hexation:

$$n \uparrow\uparrow\uparrow\uparrow x = \overbrace{n \uparrow\uparrow\uparrow n \uparrow\uparrow\uparrow n \ldots \uparrow\uparrow\uparrow n \uparrow\uparrow\uparrow n}^{x}$$

- And well we can go on forever naming bigger and bigger **hyperoperators**.

What's a hyperoperator?

- Some arithmetic operation, which follows the following rules:
$$a \uparrow^1 b = a^b$$
$$a \uparrow^n b = \overbrace{a \uparrow^{n-1} a \uparrow^{n-1} \ldots \uparrow^{n-1} a \uparrow^{n-1} a}^{b}$$
- For the purposes of this course we will use Knuth's up-arrow notation, due to it's simplicity, but keep in mind that there are other notations, such as $H_n\left(a, b\right)$ and $a[n]b$.